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Mark Scheme (Results)
Summer 2009

GCE

GCE Mathematics (6665/01)

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June 2009
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme		Marks
Q1 (a)	Iterative formula: $x_{n+1} = \frac{2}{(x_n)^2} + 2$, $x_0 = 2.5$		
	$x_1 = \frac{2}{(2.5)^2} + 2$	An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320	M1
	$x_1 = 2.32$	Both $x_1 = 2.32(0)$	A1
	$x_2 = 2.371581451...$	and x_2 = awrt 2.372	
	$x_3 = 2.355593575...$	Both x_3 = awrt 2.356	A1 cso
	$x_4 = 2.360436923...$	and x_4 = awrt 2.360 or 2.36	(3)
(b)	Let $f(x) = -x^3 + 2x^2 + 2 = 0$		
	$f(2.3585) = 0.00583577...$	Choose suitable interval for x, e.g. [2.3585, 2.3595] or tighter	M1
	$f(2.3595) = -0.00142286...$	any one value awrt 1 sf or truncated 1 sf	dM1
	Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	both values correct, sign change and conclusion	A1
		At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".	(3)
			[6]

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Question Number	Scheme	Marks
Q2 (a)	$\cos^2\theta + \sin^2\theta = 1 \quad (\div \cos^2\theta)$	
	$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ — Dividing $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ to give underlined equation.	M1
	$1 + \tan^2\theta = \sec^2\theta$	
	$\tan^2\theta = \sec^2\theta - 1$ (as required) **AG** — Complete proof. No errors seen.	A1 cso (2)
(b)	$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2, \quad (\text{eqn } *) \quad 0 \leq \theta \ll 360^\circ$	
	$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$ — Substituting $\tan^2\theta = \sec^2\theta - 1$ into eqn * to get a quadratic in $\sec\theta$ only	M1
	$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$	
	$3\sec^2\theta + 4\sec\theta - 4 = 0$ — Forming a three term "one sided" quadratic expression in $\sec\theta$.	M1
	$(\sec\theta + 2)(3\sec\theta - 2) = 0$ — Attempt to factorise or solve a quadratic.	M1
	$\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$	
	$\frac{1}{\cos\theta} = -2$ or $\frac{1}{\cos\theta} = \frac{2}{3}$	
	$\cos\theta = -\frac{1}{2}$; or $\cos\theta = \frac{3}{2}$ — $\cos\theta = -\frac{1}{2}$	A1;
	$\alpha = 120^\circ$ or $\alpha =$ no solutions	
	$\theta_1 = 120^\circ$ — 120°	A1
	$\theta_2 = 240^\circ$ — 240° or $\theta_2 = 360^\circ - \theta_1$ when solving using $\cos\theta = \ldots$	B1$\surd$
	$\theta = \{120^\circ, 240^\circ\}$ — Note the final A1 mark has been changed to a B1 mark.	(6)
		[8]

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Question Number	Scheme		Marks
Q3	$P = 80\,e^{\frac{t}{5}}$		
(a)	$t = 0 \Rightarrow P = 80e^{\frac{0}{5}} = 80(1) = \underline{80}$	$\underline{80}$	B1 (1)
(b)	$P = 1000 \Rightarrow 1000 = 80e^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = e^{\frac{t}{5}}$	Substitutes $P = 1000$ and rearranges equation to make $e^{\frac{t}{5}}$ the subject.	M1
	$\therefore\ t = 5\ln\left(\frac{1000}{80}\right)$		
	$t = 12.6286...$	awrt 12.6 or 13 years Note $t = 12$ or $t =$ awrt $12.6 \Rightarrow t = 12$ will score A0	A1 (2)
(c)	$\frac{dP}{dt} = 16e^{\frac{t}{5}}$	$ke^{\frac{1}{5}t}$ and $k \neq 80$. $16e^{\frac{1}{5}t}$	M1 A1 (2)
(d)	$50 = 16e^{\frac{t}{5}}$		
	$\therefore\ t = 5\ln\left(\frac{50}{16}\right)$ $\{= 5.69717...\}$	Using $50 = \frac{dP}{dt}$ and an attempt to solve to find the value of t or $\frac{t}{5}$.	M1
	$P = 80\,e^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80\,e^{\frac{1}{5}(5.69717...)}$	Substitutes their value of t back into the equation for P.	dM1
	$P = \frac{80(50)}{16} = \underline{250}$	$\underline{250}$ or awrt 250	A1 (3)
			[8]

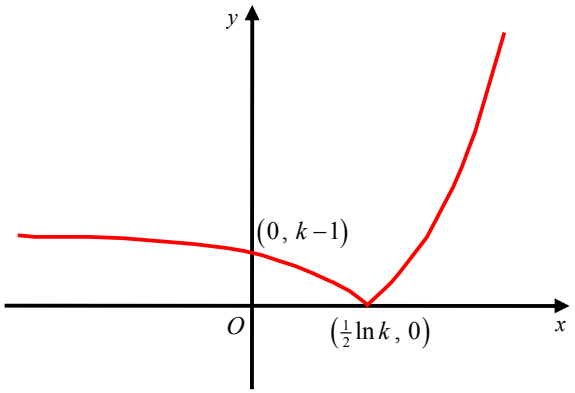
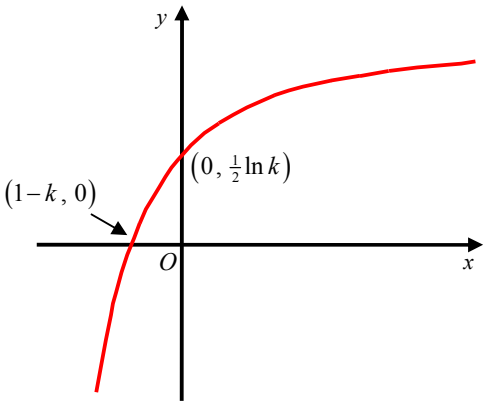
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Question Number	Scheme		Marks
Q4 (i)(a)	$y = x^2\cos 3x$		
	Apply product rule: $\left\{\begin{array}{ll} u = x^2 & v = \cos 3x \\ \frac{du}{dx} = 2x & \frac{dv}{dx} = -3\sin 3x \end{array}\right\}$		
		Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x\cos 3x \pm \beta x^2\sin 3x$	M1
	$\frac{dy}{dx} = 2x\cos 3x - 3x^2\sin 3x$	Any one term correct	A1
		Both terms correct and no further simplification to terms in $\cos\alpha x^2$ or $\sin\beta x^3$.	A1
			(3)
(b)	$y = \frac{\ln(x^2+1)}{x^2+1}$		
	$u = \ln(x^2+1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2+1}$	$\ln(x^2+1) \rightarrow \frac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \rightarrow \frac{2x}{x^2+1}$	A1
	Apply quotient rule: $\left\{\begin{array}{ll} u = \ln(x^2+1) & v = x^2+1 \\ \frac{du}{dx} = \frac{2x}{x^2+1} & \frac{dv}{dx} = 2x \end{array}\right\}$		
	$\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$	Applying $\frac{vu' - uv'}{v^2}$	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{\frac{dy}{dx} = \frac{2x - 2x\ln(x^2+1)}{(x^2+1)^2}\right\}$	{Ignore subsequent working.}	

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Question Number	Scheme		Marks
(ii)	$y = \sqrt{4x+1}, \quad x > -\frac{1}{4}$		
	At P, $y = \sqrt{4(2)+1} = \underline{\sqrt{9}} = \underline{3}$	At P, $y = \underline{\sqrt{9}}$ or $\underline{3}$	B1
	$\frac{dy}{dx} = \underline{\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)}$	$\pm k(4x+1)^{-\frac{1}{2}}$ $2(4x+1)^{-\frac{1}{2}}$	M1* A1 aef
	$\frac{dy}{dx} = \frac{2}{(4x+1)^{\frac{1}{2}}}$		
	At P, $\frac{dy}{dx} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}$	Substituting $x = 2$ into an equation involving $\frac{dy}{dx}$;	M1
	Hence m(**T**) $= \frac{2}{3}$		
	Either **T**: $y - 3 = \frac{2}{3}(x-2)$; or $y = \frac{2}{3}x + c$ and $3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3}$;	$y - y_1 = m(x-2)$ or $y - y_1 = m(x - \text{their stated } x)$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient', their x and their y_1.	dM1*;
	Either **T**: $3y - 9 = 2(x-2)$; **T**: $3y - 9 = 2x - 4$		
	T: $\underline{2x - 3y + 5 = 0}$	$\underline{2x - 3y + 5 = 0}$ Tangent must be stated in the form $ax + by + c = 0$, where a, b and c are integers.	A1
	or **T**: $y = \frac{2}{3}x + \frac{5}{3}$ **T**: $3y = 2x + 5$ **T**: $\underline{2x - 3y + 5 = 0}$		(6) **[13]**

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Question Number	Scheme	Marks
Q5 (a)	 Curve retains shape when $x > \frac{1}{2}\ln k$ Curve reflects through the x-axis when $x < \frac{1}{2}\ln k$ $(0, k-1)$ and $(\frac{1}{2}\ln k, 0)$ marked in the correct positions.	B1 B1 B1 (3)
(b)	 Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote) $(1-k, 0)$ and $(0, \frac{1}{2}\ln k)$	B1 B1 (2)
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$ Either $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$ or $\underline{f > -k}$ or $\underline{\text{Range} > -k}$.	B1 (1)
(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$ $\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \frac{1}{2}\ln(y+k) = x$ Attempt to make x (or swapped y) the subject Makes e^{2x} the subject and takes ln of both sides	M1 M1
	Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$ $\underline{\frac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$	A1 cao (3)
(e)	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k, \infty)}$ Either $\underline{x > -k}$ or $\underline{(-k, \infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer	B1 √ (1)
		[10]

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Question Number	Scheme		Marks
Q6 (a)	$A = B \Rightarrow \cos(A+A) = \cos 2A = \underline{\cos A\cos A - \sin A\sin A}$	Applies $A = B$ to $\cos(A+B)$ to give the underlined equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$	M1
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives $\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required)	Complete proof, with a link between LHS and RHS. No errors seen.	A1 **AG** (2)
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	Eliminating y correctly.	M1
	$3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$	Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.	M1
	$3\sin 2x = 2(1-\cos 2x) - 2\cos 2x$ $3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$ $3\sin 2x + 4\cos 2x = 2$	Rearranges to give correct result	A1 **AG** (3)
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x-\alpha)$ $3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$ Equate $\sin 2x$: $3 = R\sin\alpha$ Equate $\cos 2x$: $4 = R\cos\alpha$		
	$R = \sqrt{3^2+4^2}; = \sqrt{25} = 5$	$R = 5$	B1
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$	$\tan\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$ awrt 36.87	M1 A1
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$		(3)

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Question Number	Scheme		Marks
(d)	$3\sin 2x + 4\cos 2x = 2$		
	$5\cos(2x - 36.87) = 2$		
	$\cos(2x - 36.87) = \frac{2}{5}$	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$	M1
	$(2x - 36.87) = 66.42182...^\circ$	awrt 66	A1
	$(2x - 36.87) = 360 - 66.42182...^\circ$		
	Hence, $x = 51.64591...^\circ,\ 165.22409...^\circ$	One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3 Both awrt 51.6 AND awrt 165.2 If there are any EXTRA solutions inside the range $0 \le x < 180^\circ$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \le x < 180^\circ$.	A1 A1 (4)
			[12]

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Question Number	Scheme	Marks
Q7	$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}$ $x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$	
(a)	$f(x) = \frac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$ — An attempt to combine to one fraction	M1
	Correct result of combining all three fractions	A1
	$= \frac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$	
	$= \frac{x^2 + x - 12}{[(x+4)(x-2)]}$ — Simplifies to give the correct numerator. Ignore omission of denominator	A1
	$= \frac{(x+4)(x-3)}{[(x+4)(x-2)]}$ — An attempt to factorise the numerator.	dM1
	$= \frac{(x-3)}{(x-2)}$ — Correct result	A1 cso **AG**
		(5)
(b)	$g(x) = \frac{e^x - 3}{e^x - 2}$ $x \in \mathbb{R},\ x \neq \ln 2.$	
	Apply quotient rule: $\left\{ \begin{array}{ll} u = e^x - 3 & v = e^x - 2 \\ \frac{du}{dx} = e^x & \frac{dv}{dx} = e^x \end{array} \right\}$	
	$g'(x) = \frac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$ — Applying $\frac{vu' - uv'}{v^2}$	M1
	Correct differentiation	A1
	$= \frac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$	
	$= \frac{e^x}{(e^x - 2)^2}$ — Correct result	A1 **AG** cso
		(3)

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Question Number	Scheme	Marks
(c)	$g'(x) = 1 \Rightarrow \dfrac{e^x}{(e^x - 2)^2} = 1$	
	$e^x = (e^x - 2)^2$ Puts their differentiated numerator equal to their denominator.	M1
	$e^x = e^{2x} - 2e^x - 2e^x + 4$	
	$\underline{e^{2x} - 5e^x + 4} = 0$ $\underline{e^{2x} - 5e^x + 4}$	A1
	$(e^x - 4)(e^x - 1) = 0$ Attempt to factorise or solve quadratic in e^x	M1
	$e^x = 4$ or $e^x = 1$	
	$x = \ln 4$ or $x = 0$ both $x = 0,\ \ln 4$	A1
		(4)
		[12]

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Question Number	Scheme	Marks
Q8 (a)	$\sin 2x = \underline{2\sin x\cos x}$ $\underline{2\sin x\cos x}$	B1 aef (1)
(b)	$\operatorname{cosec} x - 8\cos x = 0\,, \quad 0 < x < \pi$	
	$\frac{1}{\sin x} - 8\cos x = 0$ Using $\operatorname{cosec} x = \frac{1}{\sin x}$	M1
	$\frac{1}{\sin x} = 8\cos x$	
	$1 = 8\sin x\cos x$	
	$1 = 4(2\sin x\cos x)$	
	$1 = 4\sin 2x$	
	$\underline{\sin 2x = \tfrac{1}{4}}$ $\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$ $\underline{\sin 2x = \tfrac{1}{4}}$	M1 A1
	Radians $2x = \{0.25268..., 2.88891...\}$ Degrees $2x = \{14.4775..., 165.5225...\}$	
	Radians $x = \{0.12634..., 1.44445...\}$ Degrees $x = \{7.23875..., 82.76124...\}$ Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt 0.04π or awrt 0.46π. Both $\underline{0.13}$ and $\underline{1.44}$ Solutions for the final two A marks must be given in x only. If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$.	A1 A1 cao (5) **[6]**